Comprehensive Assessment of Temporal Treatments in Crash Prediction Models

Gurdiljot Gill [1], *Wen Cheng[1*]*, *Jiao Zhou[1]*, *Tom Vo [2]*, *Xudong Jia[1]*

*1: California State Polytechnic University, Pomona; 2: Southern California Association of Governments; *: corresponding author; wcheng@cpp.edu*

Abstract

Even though a fairly large number of crash prediction models that address the serial correlations have been proposed, the comprehensive comparison of such temporal treatments under different criteria is lacking in the literature. The current study aims to fill this gap by developing nine groups of methodological approaches based on different ways of addressing temporal correlations. Moreover, three types of models were proposed for each group in terms of spatial dependency. Finally, ten different assessment criteria were utilized for the evaluation purpose. All models and performance-checking criteria applied to eight years of county-level crash counts in California.

The modeling results illustrated that the space-time models consistently enhanced the precision associated with the intercepts. The serial and spatial correlations also appeared to be statistically significant. In terms of model complexity, the models with spatial correlations outperformed the ones without considering spatially structured heterogeneity, and the models accounting for the temporal dependency revealed more benefits compared with those without temporal treatments. The opposite trends were found by prediction-pertinent criteria based on the aggregation results, even though the first-order autoregressive process space-time models with spatiotemporal interaction claimed the first place of prediction in most cases. The correlation analysis among all ten criteria illustrated that the efficiency in reducing the effective number of parameters tended to have larger impacts on the value of deviance information criterion than did the mean deviance, which demonstrated the statistically significant correlations with all other prediction-related measures.

Methodology

Bayesian Hierarchical Model Specification

$$y_{it} \sim Poisson\,(e_{it}\lambda_{it})$$

$$\begin{cases} \ln(\lambda_{it}) = \beta_0 + \xi_t + \pi_i + \upsilon_{it} + \varepsilon_{it} \\ \varepsilon_{it} \sim Normal\,(0, \tau^2) \end{cases} \quad (1)$$

Nine Groups of Temporal Treatments

Group 1: Independent-over-time Random Effect (Base)

$$\ln(\lambda_{it}) = \beta_0 + \pi_i + \upsilon_{it} + \varepsilon_{it} \quad (2)$$

Group 2: Linear Trend

$$\ln(\lambda_{it}) = \beta_0 + \beta_{t1} t + \pi_i + \upsilon_{it} + \varepsilon_{it} \quad (3)$$

Group 3: Quadratic Trend

$$\ln(\lambda_{it}) = \beta_0 + \beta_{t1} t + \beta_{t2} t^2 + \pi_i + \upsilon_{it} + \varepsilon_{it} \quad (4)$$

Group 4: Time-varying Intercepts

$$\ln(\lambda_{it}) = \beta_{0t} + \pi_i + \upsilon_{it} + \varepsilon_{it} \quad (5)$$

Group 5: First-order Autoregressive Process (AR-1)

$$\begin{cases} \varepsilon_{it} \sim normal\left(0, \frac{\sigma_{it}^2}{(1-\emptyset_1^2)}\right) \\ \varepsilon_{it} \sim normal\left(\emptyset_1 \varepsilon_{i,t-1}, \sigma_{it}^2\right), t > 1 \end{cases} \quad (6)$$

Group 6: Second-order Autoregressive Process (AR-2)

$$\begin{cases} \varepsilon_{it} \sim normal\left(0, \frac{\sigma_{it}^2}{(1-\emptyset_2)^2 - \emptyset_1^2} \cdot \frac{1-\emptyset_2}{1+\emptyset_2}\right) \\ \varepsilon_{it} \sim normal\left(\emptyset_1 \varepsilon_{i,t-1} + \emptyset_2 \varepsilon_{i,t-2}, \sigma_{it}^2\right), t > 2 \end{cases} \quad (7)$$

Group 7: First-order Moving Average Process (MA-1)

$$\begin{cases} \ln(\lambda_{it}) = \beta_0 + \pi_i + \upsilon_{it} + \varepsilon_{it} + \theta_1 \cdot \varepsilon_{i,t-1} \\ \varepsilon_{it},\ \varepsilon_{i,t-1} \sim Normal\,(0, \sigma^2) \end{cases} \quad (8)$$

Group 8: Second-order Moving Average Process (MA-2)

$$\begin{cases} \ln(\lambda_{it}) = \beta_0 + \pi_i + \upsilon_{it} + \varepsilon_{it} + \theta_1 \cdot \varepsilon_{i,t-1} + \theta_2 \cdot \varepsilon_{i,t-2} \\ \varepsilon_{it},\ \varepsilon_{i,t-1}, \varepsilon_{i,t-2} \sim Normal\,(0, \sigma^2) \end{cases} \quad (9)$$

Group 9: Time Adjacency

Similar to many other spatial correlation studies, the authors estimated temporal effects (ξ_t) in Equation 2 by applying CAR (conditional autoregressive) formulation which incorporates the temporal correlation among crashes occurring at neighboring years. The details of CAR can be found in the following subsection. It is important to note that the relatively small sample size (t=8) might harm the performance of this group to some degree.

Three Types of Models

Type 1: Temporal Correlation only

$$\ln(\lambda_{it}) = \beta_0 + \xi_t + \varepsilon_{it} \quad (10)$$

Type 2: Space-Time Model without Spatiotemporal Interactions

$$\ln(\lambda_{it}) = \beta_0 + \xi_t + \pi_i + \varepsilon_{it} \quad (11)$$

Type 3: Space-Time Model with Spatiotemporal Interactions

$$\ln(\lambda_{it}) = \beta_0 + \xi_t + \pi_i + \delta_i \cdot \boldsymbol{T} + \varepsilon_{it} \quad (12)$$

Evaluation Criteria

Criteria 1~3: DIC and Its Components

$$\begin{cases} DIC = \bar{D} + P_D \\ D(\theta) = -2 \cdot \log(p(y|\theta)) \\ \bar{D} = E^{\theta}[D(\theta)],\ \ P_D = \bar{D} - D(\bar{\theta}) \end{cases} \quad (15)$$

Criterion 4: Log Pseudo Marginal Likelihoods (LPML)

$$\begin{cases} CPO = \left(\frac{1}{T}\sum_{t=1}^{T} \frac{1}{p(Y_i|\beta^{(t)})}\right)^{-1} \\ LPML = \sum_{i=1}^{n} \log(CPO_i) \end{cases} \quad (16)$$

Criterion 5: Mean Absolute Deviation (MAD)

$$MAD = \frac{1}{n}\sum_{i=1}^{n} |Y_i - O_i| \quad (17)$$

Criterion 6: Mean Square Predictive Error (MSPE)

$$MSPE = \frac{1}{n}\sum_{i=1}^{n} (Y_i - O_i)^2 \quad (18)$$

Criterion 7: The R_p^2 statistic

$$R_p^2 = 1 - \frac{\sum_{i=1}^{n}\left[\frac{O_i - Y_i}{\sqrt{Y_i}}\right]^2}{\sum_{i=1}^{n}\left[\frac{O_i - \bar{O}}{\sqrt{\bar{O}}}\right]^2} \quad (19)$$

Criterion 8: The G^2 statistic

$$G^2 = 2\sum_{i=1}^{n} O_i LN\left(\frac{O_i}{Y_i}\right) \quad (20)$$

Criterion 9: Chi-squared Residual Sum of Squares (RSS)

$$RSS = \sum_{i=1}^{n} \frac{(O_i - Y_i)^2}{Y_i} \quad (21)$$

Criterion 10: Total Rank Difference (TRD)

$$TRD = \sum_{i=1}^{n} |R(i_o) - R(i_Y)| \quad (22)$$

Evaluation Results

Group	Type	$\bar{D}$	P_d	DIC	LPML	MAD	MSPE	R_p^2	G^2	RSS	TRD
1	1	4328	443.5	4771	-2090.0	2.611	**10.814**	0.994	27.2	37.1	382
	2	4342.3	426.4	4768	-2094.4	4.755	42.192	0.989	35.4	68.5	571
	3	4328.5	465.3	4793	-2088.6	2.284	18.359	0.998	22.2	12.3	216
2	1	4331.3	442.2	4773	-2091.1	2.684	11.780	0.993	11.3	41.7	399
	2	4360.1	392.4	4752	-2102.2	5.974	76.870	0.982	102	116.7	862
	3	4369.6	393.4	4763	-2106.2	5.992	76.448	0.982	111	116.2	862
3	1	4331.5	442.3	4773	-2091.2	2.685	11.763	0.993	26.8	41.8	402
	2	4358.9	389.3	4748	-2102.0	6.119	80.481	0.982	99.8	118.5	864
	3	4373.0	390.1	4763	-2106.0	6.202	87.479	0.98	182.1	130.5	918
4	1	4331.2	442.2	4773	-2091.0	2.697	11.855	0.993	61.1	41.1	401
	2	4358.3	388.6	4746	-2101.9	6.263	83.228	0.981	112.7	119.0	871
	3	4370.4	387.7	4758	-2105.4	6.327	90.072	0.98	70.9	131.3	908
5	1	4350.7	450.3	4801	-2092.0	2.685	14.752	0.994	9.3	39.5	383
	2	4355.1	420.4	4775	-2094.2	5.464	57.852	0.989	102	72.6	611
	3	**4322.8**	451.2	4774	**-2084.2**	**1.858**	15.378	**0.998**	53.1	**6.9**	**163**
6	1	4491.1	**332.7**	4823	-2142.7	16.41	2727.5	0.956	274.1	286.6	1097
	2	4397.9	337.3	**4735**	-2110.2	9.429	215.03	0.971	131.6	188.4	1079
	3	4388.4	346.9	**4735**	-2106.8	8.865	193.64	0.974	155.9	167.8	1033
7	1	4346.1	413.4	4759	-2091.7	3.897	29.842	0.99	26.2	65.3	586
	2	4384.1	394.3	4778	-2103.2	6.242	82.645	0.982	117.8	117.9	821
	3	4365.8	429.9	4795	-2095.1	4.370	44.389	0.991	55.7	58.2	577
8	1	4361.6	398.4	4760	-2092.4	4.342	41.117	0.987	**-8.4**	80.6	676
	2	4405.6	386.7	4792	-2105.4	6.405	93.739	0.979	108	132.3	861
	3	4377.7	434.5	4812	-2094.3	3.967	39.525	0.991	27	58.7	563
9	1	4330.7	442.3	4773	-2090.9	2.690	11.878	0.993	96.3	40.6	394
	2	4354.1	385.7	**4739**	-2100.9	6.161	81.223	0.982	62	118.1	867
	3	4364.8	382.2	4747	-2104.1	6.277	88.442	0.98	130.1	130	902

Notes: 1. The shaded cells indicate the worst performance of models under different evaluation criteria.
2. The bold fonts indicate the best performance of models under different evaluation criteria.

Conclusion

The posterior model estimates demonstrated the notable impact of inclusion of spatial correlation on improving the precision of intercept across all groups of temporal treatments. Both spatial and temporal random effects were noted to be statistically significant reflecting the presence of spatial clustering and temporal dependency of crash counts which accounted for the variability in models. The changing coefficients of intercept for the random intercept model further demonstrated the impact of serial changes on crash risk.